Inter-Landau-level skyrmions versus quasielectrons in the $\nu = 2$ quantum Hall effect

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We consider the charged excitations in the quantum Hall effect with a large Zeeman splitting at filling factor $\nu = 2$. When the Zeeman splitting is increased over a critical value the ground state undergoes a first order phase transition from a paramagnetic to a ferromagnetic phase. We have studied the possibility of forming charged spin-textured excitations, "inter-Landau-level skyrmions," close to this transition, and find that these never are the lowest lying charged excitations but can under certain conditions provide an effective driving mechanism for the paramagnetic to ferromagnetic phase transition. We show that the charged excitations inevitably present in the system can act as nucleation centers even at $T = 0$ and hence set a specific limit for the maximal hysteresis attainable in this case. Calculations of how the finite width of the two-dimensional electron gas affects the $\nu = 2$ phase transition and the polarization at higher filling factors are also presented.

I. INTRODUCTION

Ferromagnetic quantum Hall systems are known to have charged excitations, skyrmions, that involve texturing of spin. So far, these skyrmions have been considered only at odd integer filling factors and fractional filling factors smaller than 1. Skyrmions were predicted to be the lowest energy charged excitations in, for example, GaAs at filling factor $\nu = 1$.[1,2] This was later confirmed in several experiments.[3–6] Under some conditions skyrmions are also predicted to form at higher odd filling factors[7,8] and there are experiments claiming to have seen this.[9]

Since skyrmions involve moving particles to unoccupied spin-flipped states, they can be candidates for the lowest energy excitations only when there is a small single-particle gap for such spin flips. In most materials spin-orbit coupling decreases the effective Landé factor g of the electrons—in for example GaAs g is effectively reduced from 2 to -0.44. At the same time the cyclotron gap $\hbar\omega_c$ is normally increased due to the small effective mass of the conducting electrons in these materials. This means that the Zeeman energy $g\mu_B B$ will normally be small compared to the cyclotron gap $\hbar\omega_c$. Hence small single-particle gaps will be found at odd filling factors.

There are, however, materials where spin-orbit coupling is so strong that the magnitude of the Zeeman energy is instead strongly enhanced. In InSb the effective Landé factor g is of the order of -50.[10] The ratio of the Zeeman energy to the cyclotron energy gap can also, as always, be increased by tilting the sample relative to the magnetic field. This makes it possible to reach a limit where the spin-split single-particle energy levels from the lowest and next lowest orbital Landau levels can come very close together or even cross (see Fig. 1). In this case there is a small single-particle gap at *even* filling factors and one can imagine the possibility of having "inter-Landau-level" skyrmions—these would be charged excitations that involve spins flipped from one orbital Landau level to another.

In this paper we first review the paramagnetic to ferromagnetic phase transition that occurs at filling factor $\nu = 2$ when single-particle levels from different orbital Landau levels come close. We present finite width calculations of how the Coulomb interaction affects this transition and comment on recent experiments at higher filling factors by Papadakis *et al.*[13] on the spin splitting in AlAs quantum wells. We then investigate the charged excitations in the possible $\nu = 2$ quantum Hall states and find that inter-Landau-level skyrmions are never the lowest energy charged excitations in this system, but do provide an effective driving mechanism for the phase transition and thereby limit the maximal hysteresis attainable in this case.

II. THE PARAMAGNETIC TO FERROMAGNETIC TRANSITION

In the limit when the Coulomb energy is negligible compared to the gaps in the single-particle spectrum, we can specify the groundstate of a quantum Hall system at integer filling factors by giving the order in which the single-particle energy levels are filled. For finite but

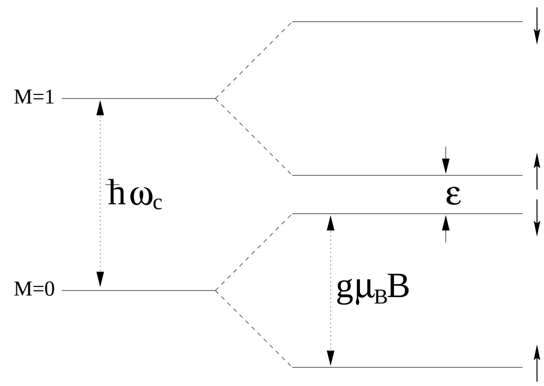

FIG. 1. The single-particle energy spectrum. Here the Zeeman gap $g\mu_B B$ and the cyclotron gap $\hbar\omega_c$ are comparable in magnitude, leaving a small gap ϵ between the second and third levels.

small Coulomb interaction, spin polarized states will be favored and the order in which levels are filled is determined by a competition between single-particle energies and the Coulomb energy.

In the case when the cyclotron gap $\hbar\omega_c$ is larger than both the Zeeman energy $g\mu_B B$ and the characteristic Coulomb energy $e^2/\epsilon\ell$, the $\nu = 2$ ground state is simply the lowest orbital Landau level with both spin states filled. This state is paramagnetic with no net polarization. Now if the Zeeman energy is increased the system will undergo a first order phase transition to the ferromagnetic state with the same spin filled in the two lowest Landau levels. The Coulomb interaction will cause this transition to occur *before* the single-particle energies cross.[11,12] Evaluating the Coulomb energy in the paramagnetic state $E_C^{\rm PM}$ and the ferromagnetic state $E_C^{\rm FM}$ yields

$$\begin{aligned} E_C^{\rm PM} &= 2E_{00}, \\ E_C^{\rm FM} &= E_{00} + E_{11} + 2E_{01}. \end{aligned} \tag{1}$$

Here E_{MN} is the total exchange energy contribution of the filled level M to level N,

$$\begin{aligned} E_{MN} = &-\tfrac{1}{2}\sum_{m,n}\int d^2r\int d^2r' V(|\mathbf{r}-\mathbf{r}'|)\times \\ &\phi^*_{Mm}(\mathbf{r})\phi_{Nn}(\mathbf{r})\phi^*_{Nn}(\mathbf{r}')\phi_{Mm}(\mathbf{r}')\,, \end{aligned} \tag{2}$$

where ϕ_{Mm} is the wave function with (angular) momentum m in Landau level M. In the ideal (zero thickness) two-dimensional case E_{MN} can be evaluated exactly,

$$E_{MN} = -N_\phi \frac{1}{\sqrt{2}}\int_0^\infty dte^{-t^2}L_M(t^2)L_N(t^2), \tag{3}$$

where L_M are the Laguerre polynomials and N_ϕ is the number of flux quanta in the system, i.e., the number of particles in each level. The difference in Coulomb energy between the two states, $E_C^{\rm PM} - E_C^{\rm FM}$, in this ideal two-dimensional (2D) case is $\frac{3}{8}\sqrt{\pi/2}N_\phi$; this means that the ferromagnetic state will have the lowest total energy when

$$\epsilon \equiv \hbar\omega_c - g\mu_B B \le \epsilon_c = \frac{3}{8}\sqrt{\frac{\pi}{2}} \approx 0.470, \tag{4}$$

where energies here and onward are given in units of $e^2/\epsilon\ell$. This instability may seem large but is drastically reduced when the finite width of the 2D electron gas is taken into account. Using a Gaussian subband approximation[7] we calculate ϵ_c numerically for different effective widths w. Results are given in Table I where w is given in units of the magnetic length. The finite width dependence of ϵ_c closely follows the form

$$\epsilon_c(w) = \frac{a}{w+b}, \tag{5}$$

where $a \approx 0.170$ and $b \approx 0.359$.

w	0	0.5	1.0	2.0
ϵ_c	0.4700	0.2010	0.1227	0.0673

TABLE I. Finite width dependence of ϵ_c.

It is worth noting that for small enough densities (and hence small magnetic fields) the ferromagnetic $\nu = 2$ state has the lowest energy even for a vanishing Zeeman energy. This happens because the Coulomb energy—proportional to $\sqrt{B}$—is larger than the cyclotron energy—proportional to B—for small enough B. Again the critical density for this ferromagnetic ordering depends strongly on the width of the 2D gas. We can easily predict it for some common materials, now keeping the Zeeman energy at zero tilt angle at $\nu = 2$; see Table II.

	w			
	0	0.5	1.0	2.0
InSb	4.8×10^9	8.8×10^8	3.3×10^8	9.9×10^7
GaAs	7.5×10^{10}	1.4×10^{10}	5.1×10^9	1.5×10^9
AlAs	2.4×10^{13}	4.4×10^{12}	1.6×10^{12}	4.9×10^{11}

TABLE II. Critical densities (in cm^{-2}) below which the $\nu = 2$ ground state will be polarized at zero tilt angle for different widths of the 2D electron gas.

In GaAs and InSb these critical densities are too low to be experimentally relevant. In AlAs we use an effective mass of $m^* = 0.46m_e$ corresponding to the 2D electron gas occupying ellipsoidic constant-energy surfaces with one major axis in the plane of the 2D interface and find a critical density that is larger than that used in experiments. This is compatible with experiments by Papadakis *et al.*[13] who report on measurements where they see a fully polarized $\nu = 3$ state at zero tilt angle in this system. We must note that, since the Coulomb energy normally is larger than the cyclotron gap in AlAs, the true ground state will contain a large mixture of different Landau levels and the simple analysis here is not expected to be precise. Nevertheless, the same method applied to the $\nu = 3$ ground state with a physical width of 150 Å (as used in the setup of Papadakis *et. al.*) yields a critical density of 3.8×10^{11} cm^{-2} below which the $\nu = 3$ groundstate should be fully polarized. This agrees qualitatively with the range of densities used in this experiment [$(1.4$–$3.9)\times10^{11}$ cm^{-2}].

III. THE CHARGED EXCITATIONS

We have used a time dependent Hartree-Fock method to look for spin-flip instabilities around a quasielectron or quasihole in both the paramagnetic and ferromagnetic $\nu = 2$ ground states. This instability analysis is equivalent to taking the small spin limit of an inter-Landau-level

skyrmion and will tell us whether or not the charged excitations involve extra flipped spins. Before describing the calculation we want to note that Kohn's theorem,[14] stating that electron-electron interactions cannot affect the cyclotron resonance in a translationally invariant system, does not imply anything for the excitations in this case since Kohn's theorem does not apply when the excited state has a different spin from the ground state.

In the paramagnetic case we write the ground state as

$$| \psi_0\rangle_{\rm PM} = \prod_{m=0} c^\dagger_{0,m,\downarrow} c^\dagger_{0,m,\uparrow} \mid 0\rangle, \tag{6}$$

where $c^\dagger_{M,m,\sigma}$ creates an electron in Landau level M with angular momentum m and spin σ. We use the symmetric gauge where m takes integer values from $-M$ and upward. Next we create a charged excitation (a quasihole) at the origin by removing one electron from the upper level. To find out if further spin flips are favored we allow an inter Landau level spin wave around the hole,

$$\Psi^\dagger(q)\ c_{0,0,\downarrow} \mid \psi_0\rangle_{\rm PM}, \tag{7}$$

where $\Psi^\dagger(q)$ is a spin wave operator that flips a spin from the $M = 0$ spin-down level to the $M = 1$ spin-up level with change q in momentum,

$$\Psi^\dagger\,(q) = \sum_{k=1} \alpha_k c^\dagger_{1,k+q,\uparrow} c_{0,k,\downarrow}. \tag{8}$$

We determine the α_k's and the corresponding energies by numerically diagonalizing the Coulomb Hamiltonian in this subspace for each q. Note that q can take values from -2 and upward here. For $q = -2$ we find a negative eigenvalue of -0.269 in the ideal 2D case. This means that the charged excitations will involve extra spin flips and form a charged spin-texture excitation—a skyrmion—if the single-particle gap to the next level is smaller than this instability, i.e., $\epsilon \le 0.269e^2/\epsilon\ell$. However, this value is smaller than the instability to the ferromagnetic state described in the previous section ($\epsilon_c = 0.470$); thus the system will in this case undergo the transition to the fully polarized state *before* skyrmions become the preferred quasiparticles.

Since this is a question of energetics it is not guaranteed that the same conclusion holds for finite widths of the 2D electron gas. Indeed, at filling factor $\nu = 3$ skyrmions are predicted to form for finite widths but not in the ideal 2D case.[7,8] In the present case, however, the two instabilities both decrease at roughly the same rate and the polarized electron/hole quasiparticle is favored up to a width of at least five magnetic lengths.

In the ferromagnetic state,

$$| \psi_0\rangle_{\rm FM} = \prod_{m=0,n=-1} c^\dagger_{1,n,\uparrow} c^\dagger_{0,m,\uparrow} \mid 0\rangle, \tag{9}$$

the system can *gain* single-particle energy by flipping one spin from the filled $M = 1$ spin-up level to the empty $M = 0$ spin-down level below it. By first diagonalizing the Coulomb Hamiltonian of this spin wave with no hole present, we confirm that any such spin flip always costs more than the maximum possible single-particle gain—the lowest energy spinwave costs $0.51e^2/\epsilon\ell$. Hence the ground state is stable and the phase transition is first order as promised. We then redo the diagonalization with a quasihole present and find that the spin wave is unaffected by the localized excitation, so in this case the spin wave again costs more Coulomb energy than the potential single-particle gain. Hence there is no situation where a skyrmion can have lower energy than an electron/hole quasiparticle in this case either.

Since the transition between the paramagnetic and ferromagnetic phase is first order, it should be possible to "supercool" the system and stay in the metastable state. One could imagine, for example, starting in the stable paramagnetic phase and then slowly increase the Zeeman splitting while keeping the filling factor fixed close to $\nu = 2$. This could be realized by rotating the sample *in situ* while increasing the total magnetic field. When the single-particle gap ϵ falls below ϵ_c ($= 0.47$ in the 2D case) the paramagnetic state ceases to be the true ground state. However, there may still be no effective way for the system to overcome the barrier separating the two phases, in which case it will stay in the metastable paramagnetic phase for a very long time. In a region below ϵ_c the barrier is large since a single spin flip still costs a large energy (even though we know that flipping *all* spins would take us to the true ground state). Naively, one could hope to form skyrmions by supercooling the paramagnetic state beyond the point where the skyrmion instability sets in (i.e., $\epsilon < 0.269$). But since forming a skyrmion does involve flipping spins it seems likely that instead of forming skyrmions in the metastable paramagnetic state this spin-flip instability rather provides an effective way for the system to undergo the phase transition and actually drives it to the true (ferromagnetic) ground state. Note that this instability will be present even at $T = 0$ since there will always be a finite density of charged excitations in the system (except at the exact center of the $\nu = 2$ plateaux). Once the instability sets in, these charged excitations will act as nucleation centers for the other phase and thus this instability sets an upper limit for how large a hysteresis one can obtain when approaching $T = 0$.

We can also note that the hysteresis is asymmetric in the sense that the paramagnetic state can survive as metastable further into the ferromagnetic region than the ferromagnetic state can into the paramagnetic region (the hysteresis is limited to $0.269 < \epsilon < 0.51$, with $\epsilon_c = 0.47$ rather close to the upper of these limits).

IV. CONCLUSION

We have investigated the nature of the charged excitations in a quantum Hall system with large Zeeman energy at filling factor $\nu = 2$, and find that there is no instability to flip extra spins around the polarized quasiparticles either in the paramagnetic or in the ferromagnetic ground state. Hence no inter-Landau-level skyrmions can be the lowest energy charged excitations in this system. The skyrmion instability does, however, limit the possible hysteresis attainable in the paramagnetic to ferromagnetic phase transition. We also presented finite thickness calculations for the first order phase transition between the paramagnetic and the ferromagnetic ground state and predict some critical densities below which the ferromagnetic state has lowest energy even for zero tilt angle.

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